MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 31-08-2021 (MORNING SHIFT) | TIME : (9.00 am to 12.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Angular momentum of a single particle moving with constant speed along circular path

(1) Changes in magnitude but remains same in the direction

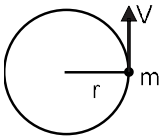
(2) is zero

(3) remains same in magnitude and direction

(4) remains same in magnitude but changes in the direction

Ans. (3)

Sol.



$\vec{L}$ $\vec{r}$ $\vec{p}$ $\vec{r}$ $m\vec{v}$

2. A body of mass M moving at speed V_0 collides elastically with a mass 'm' at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is :

(1) 1 (2) 4 (3) 2 (4) 3

Ans. (4)

Sol.

Linear momentum conservation

(i) y-direction

$mv_2\sin\theta = mv_1\sin\theta$

$mv_2 = mv_1$ .....(1)

(ii) x-direction

$mv_0 = mv_2\cos\theta + mv_1\cos\theta$ .....(2)

from (1) and (2)

$mv_0 = mv_2\cos\theta + mv_2\cos\theta$

$mv_0 = 2mv_2\cos\theta$ .....(3)

(iii) e = 1 given

$$\frac{v_2 \quad v_1 \cos 2}{v_0 \cos} \quad 1$$

$v_2 - v_1\cos2\theta = v_0\cos\theta$ .....(4)

$$v_2 \quad \frac{mv_2}{M}\cos 2 \quad v_0 \cos$$

$$v_2 \quad 1 \quad \frac{m}{M}\cos 2 \quad v_0 \cos$$

$$\frac{mv_0}{2m\cos} \quad 1 \quad \frac{m}{M}\cos 2 \quad v_0 \cos$$

$M - m\cos2\theta = 2m\cos2\theta$;

$M = 2m\cos^2\theta = m(2\cos^2\theta - 1)$

$$\frac{M}{m} \quad 4\cos^2 \quad 1$$

$$\frac{M}{m}_{max} \quad 4 \quad 1 \quad 3$$

3. In an ac circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is :

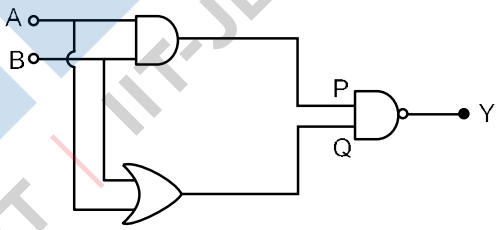
(1) $R\sqrt{2}$ (2) R (3) $2R^2$ (4) Zero

Ans. (2)

Sol. $Z \quad \sqrt{(X_L \quad X_C)^2 \quad R^2}$

$Z = R$

4. In the following logic circuit the sequence of the inputs A, B are (0, 0), (0, 1), (1, 0) and (1, 1). The output Y for this sequence will be:



(1) 0, 0, 1, 1 (2) 1, 0, 1, 0 (3) 0, 1, 0, 1 (4) 1, 1, 1, 0

Ans. (4)

Sol.

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

5. A reversible engine has an efficiency of $\frac{1}{4}$. If the temperature of the sink is reduced by 58°C, its efficiency becomes double. Calculate the temperature of the sink:

(1) 180.4°C (2) 382°C (3) 174°C (4) 280°C

Ans. (3)

Sol. $1 \quad \frac{T_L}{T_H}$

$\frac{1}{4} \quad 1 \quad \frac{T_L}{T_H}$

$\frac{T_L}{T_H} \quad \frac{3}{4}$

$2 \quad \frac{1}{2} \quad 1 \quad \frac{T_L \quad 58}{T_H}$

$\frac{T_L \quad 58}{T_H} \quad \frac{1}{2}$

$\frac{T_L}{T_L \quad 58} \quad \frac{3/4}{1/2}$

$2\,T_L = 3T_L - 174$

$T_L = 174°C$

6. Match List-I with List-II.

	List-I	List-II
(a)	Torque	(i) MLT^{-1}
(b)	Impulse	(ii) MT^{-2}
(c)	Tension	(iii) ML^2T^{-2}
(d)	Surface	(iv) MLT^{-2}

Choose the most appropriate answer from the option given below:

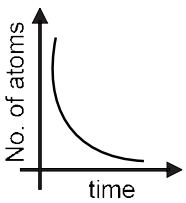
(1) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii) (2) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(3) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii) (4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

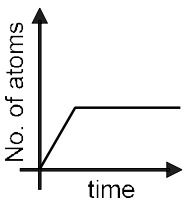
Ans. (4)

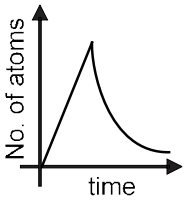
7. A sample of a radioactive nucleus A disintegrates to another radioactive nucleus B, which in turn disintegrates to some other stable nucleus C. Plot of a graph showing the variation of number of atoms of nucleus B versus time is : (Assume that at t = 0, there are no B atoms in the sample)

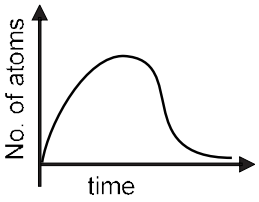
(1)



(2)



(3) 

(4) 

Ans. (4)

Sol. $A \xrightarrow{\lambda} B \xrightarrow{\lambda} C$

t = 0	N_0	O	O
t = t	$N_0 e^{-\lambda t}$	N	N_C

Where λ = decay constant

$\frac{dN}{dt}$ decay rate of A – decay rate of B

$= N_0 e^{-\lambda t} - \lambda N$

$\frac{dN}{dt}$ slope of N v/s t graph

$\frac{d^2N}{dt^2}$ $N_0 e^{t}(\quad)$ $[N_0 e^{t} \quad 1]$

$\frac{d^2N}{dt^2}$ ve so graph is (a)

8. An object is placed at the focus of concave lens having, focal length f. What is the magnification and distance of the image from the optical centre of the lens?

(1) 1/2, f/2 (2) 1/4, f/4 (3) 1, ∞ (4) very high, ∞

Ans. (1)

Sol. $\frac{1}{F} \quad \frac{1}{v} \quad \frac{1}{u}$

Put u = – f F = – f

$\frac{1}{f} \quad \frac{1}{v} \quad \frac{1}{f}$; $\frac{1}{f} \quad \frac{1}{f} \quad \frac{1}{v}$; $\frac{2}{f} \quad \frac{1}{v}$ v $\frac{f}{2}$

Magnification = v/u $\frac{f}{2(\ f)}$ $\frac{1}{2}$

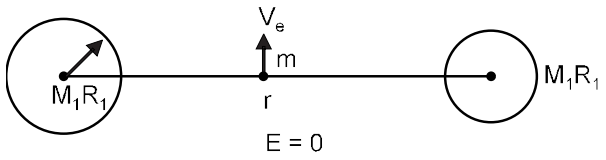
9. The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centers are at a distance r apart. Find the minimum escape velocity for a particle of mas m to be projected from the middle of these two masses :

(1) V $\frac{1}{2}\sqrt{\frac{2G(M_1 \quad M_2)}{r}}$ (2) V $\sqrt{\frac{4G(M_1 \quad M_2)}{r}}$

(3) V $\frac{\sqrt{2G}(M_1 \quad M_2)}{r}$ (4) V $\frac{1}{2}\sqrt{\frac{4G(M_1 \quad M_2)}{r}}$

Ans. (2)

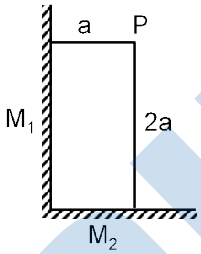
Sol.



$$\frac{GM_1}{r/2}\ \frac{GM_2}{r/2}\ m\ \frac{1}{2}mv_e^2\ \ 0$$

$$V_e\ \sqrt{\frac{4G(M_1\ \ M_2)}{r}}$$

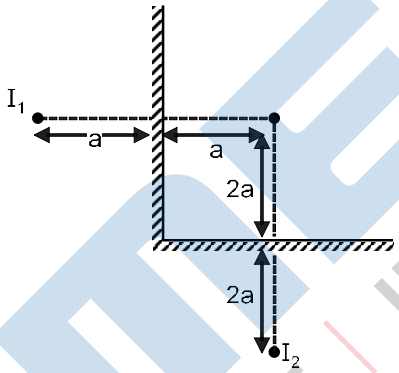
10. Two plane mirror M_1 and M_2 are at right angle to each other shown. A point source 'P' is placed at 'a' and '2a' meter away from M_1 and M_2 respectively. The shortest distance between the images thus formed is: (Take $\sqrt{5}\ \ 2.3$)



(1) 4.6a (2) $2\sqrt{10}\,a$ (3) 3a (4) 2.3a

Ans. (1)

Sol.



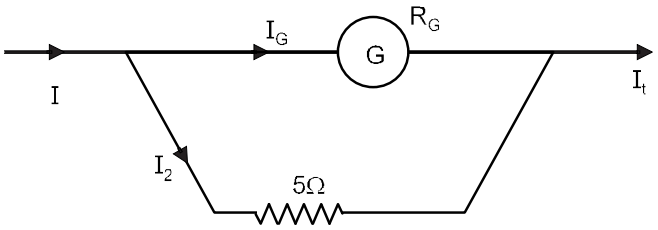
Distance between I_1 and $I_2\ \sqrt{(4a)^2\ \ (2a)^2}\ \ 4.6a$

11. Consider a galvanometer shunted with 5Ω resistance and 2% of current passes through it. What is the resistance of the given galvanometer?

(1) 226 Ω (2) 245 Ω (3) 344 Ω (4) 300 Ω

Ans. (2)

Sol.



$I_G \quad \frac{2}{100} \quad I$ (Given)

$I_G = 5\Omega \times I$

$(R_G + 5\Omega)$

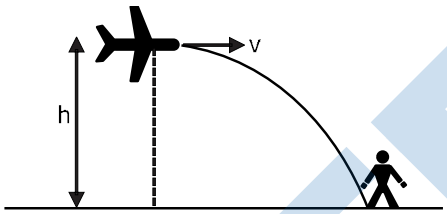
$\frac{5}{R_G \quad 5} \quad \frac{1}{50} \Rightarrow 250 = R_G + 5 \Rightarrow R_G = 245\ \Omega$

12. A helicopter is flying horizontally with a speed v at an altitude h has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped ?

(1) $\sqrt{\frac{2ghv^2 \quad 1}{h^2}}$ (2) $\sqrt{2ghv^2 \quad h^2}$ (3) $\sqrt{\frac{2gh}{v^2}} \quad h^2$ (4) $\sqrt{\frac{2v^2h}{g} \quad h^2}$

Ans. (4)

Sol.



The aeroplane imparts v velocity to the food packet horizontally when dropping.

Horizontal range of food packet $v \sqrt{\frac{2h}{g}}$

Also, Horizontal distance, travelled by aeroplane $v \sqrt{\frac{2h}{g}}$

thus, vertical distance between aeroplane and person = h

distance $\sqrt{\frac{2v^2h}{g} \quad h^2}$

13. A moving proton and electron have the same de-Broglie wavelength. If K and P denote the K.E. and momentum respectively. Then choose the correct option :

(1) $K_p < K_e$ and $P_p = P_e$ (2) $K_p > K_e$ and $P_p = P_e$

(3) $K_p = K_e$ and $P_p = P_e$ (4) $K_p < K_e$ and $P_p < P_e$

Ans. (1)

Sol. $_{deb} \quad \frac{h}{p} \quad \frac{h}{\sqrt{2m\ K.E.}}$

If λ is same the P will be same

KE $\propto$ 1/m

$m_p > m_e \Rightarrow K_p < K_e$

14. For an ideal gas the instantaneous change in pressure 'p' with volume 'v' is given by the equation dp/dv = –ap. If $p = p_0$ at v = 0 is the given boundary condition, then the maximum temperature one mole of gas can attain is : (Here R is the gas constant)

(1) $\frac{ap_0}{eR}$ (2) $\frac{p_0}{aeR}$ (3) infinity (4) 0°C

Ans. (2)

Sol. $\frac{dP}{dV} \quad aP$

$\int_{P_0}^{P} \frac{dP}{P} \quad \int_{0}^{V} adV \quad \ln\frac{P}{P_0} \quad aV$

$P = P_0e^{-aV}$

Now, PV = RT (as n = 1)

$P \quad V\frac{dP}{dV} \quad R\frac{dT}{dV}$ [for maximum T, $\frac{dT}{dV} \quad 0$]

$P \quad aPV \quad 0 \quad V \quad \frac{1}{a}.$

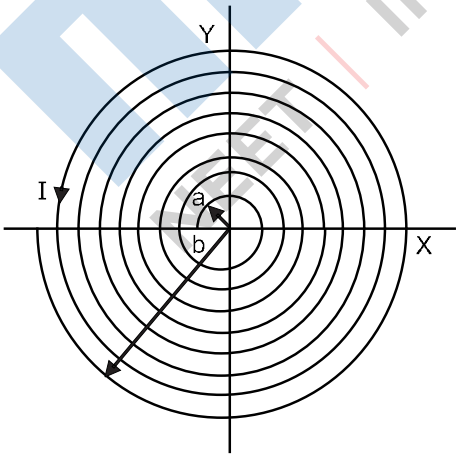
Thus, $T \quad \frac{PV}{R} \quad \frac{P_0e^{\ 1}}{Ra} \quad \frac{P_0}{eRa}$

15. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii 'a' and 'b' respectively. Find the magnetic field at centre, when a current I passes through coil :

(1) $\frac{_0I}{a(a \quad b)} \quad \frac{1}{a} \quad \frac{1}{b}$ (2) $\frac{_0IN}{2(a \quad b)}\log_e \frac{b}{a}$ (3) $\frac{_0I}{8} \quad \frac{a \quad b}{a \quad b}$ (4) $\frac{_0I}{8} \quad \frac{a \quad b}{a \quad b}$

Ans. (2)

Sol.



$$B \quad \frac{{}_0 dNi}{2x} \quad \frac{{}^{0}\ \frac{N}{b\ \ a}dx\ \ i}{2x} \quad \frac{{}_0 Ni}{2(b\ \ a)}\ell n\frac{b}{a}$$

16. A uniform heavy rod of weight 10 kg ms^{-2}, cross-sectional area 100 cm^2 and length 20 cm is hanging from a fixed support. Young modulus of the material of the rod is 2×10^{11} Nm^{-2}. Neglecting the lateral contraction, find the elongation of rod due to its own weight :

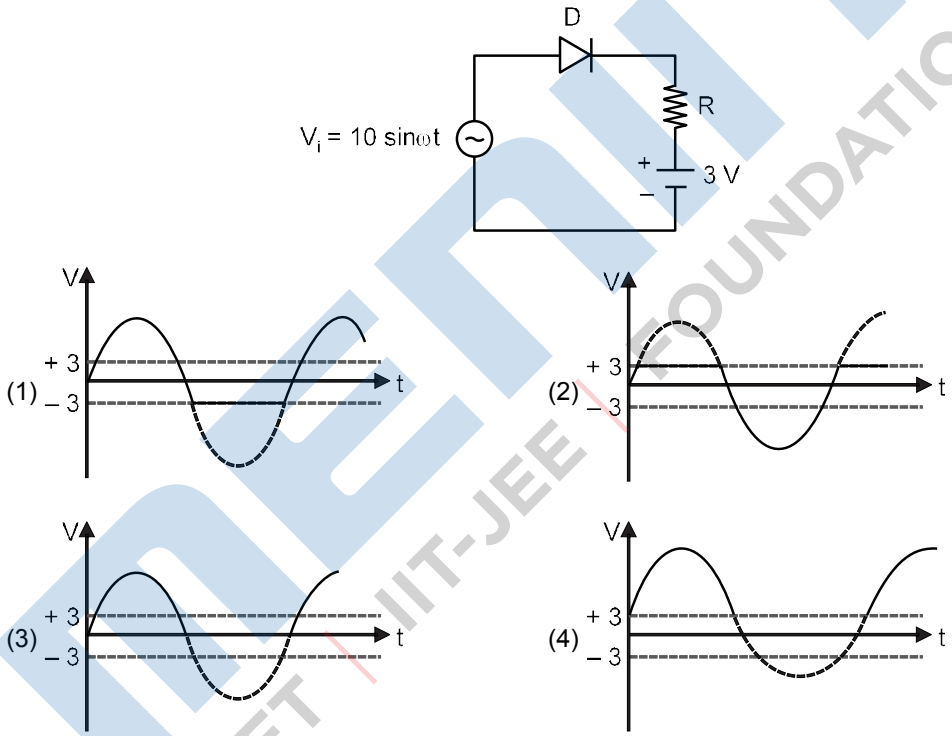
(1) 4×10^{-8} m (2) 5×10^{-10} m (3) 2×10^{-9} m (4) 5×10^{-8} m

Ans. (2)

Sol. Elongation due to self weight $\frac{w\ |}{2YA}$

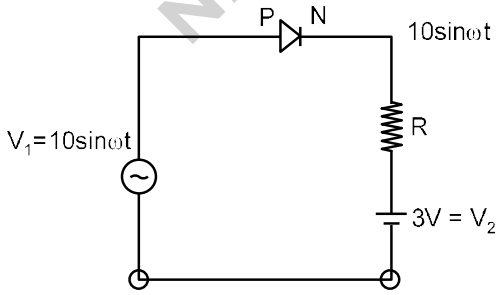
$$\frac{10\ \ 0.2}{2\ \ 2\ \ 10^{11}\ \ 100\ \ 10^{\ 4}}\ \ 5\ \ 10^{\ 10}\ m$$

17. Choose the correct waveform that can represent the voltage across R of the following circuit, assuming the diode is ideal one:



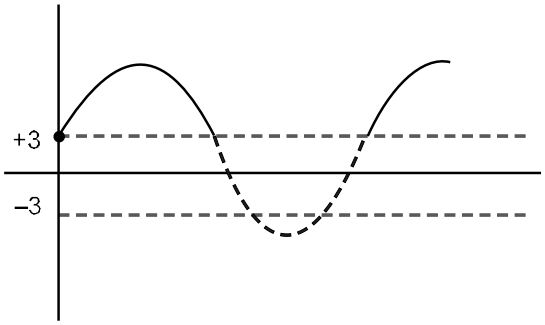
Ans. (4)

Sol.

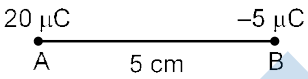


Current will flow from the diode when diode is in forward biased.

Thus $\Delta V = (10 \sin\omega t - 3)$ V and ΔV will be 'zero', when diode is reverse biased.



18. Two particles A and B having charges 20 μC and –5 μC respectively are held fixed with a separation of 5 cm. At what position a third charged particle should be placed so that it does not experience a net electric force?



(1) At midpoint between two charges

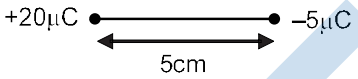
(2) At 1.25 cm from a – 5 μC between two charges

(3) At 5 cm from 20 μC on the left side of system

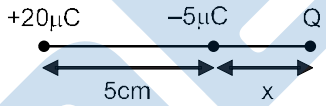
(4) At 5 cm from –5 μC on the right side

Ans. (4)

Sol.



Charge should be placed near – 5 μC at x distance



$$\frac{K(20)Q}{(5+x)^2} \quad \frac{K(5)Q}{X^2}$$

$4x^2 = (5+x)^2$

$4x^2 = 25 + x^2 + 10x$

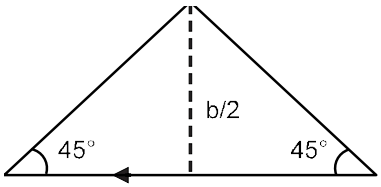
$3x^2 - 10x - 25 = 0 \quad \Rightarrow \quad x = 5$

19. A small square loop of side 'a' and one turn is placed inside a larger square loop of side b and one turn (b > > a). The two loops are coplanar with their centres coinciding. If a current I is passed in the square loop of side 'b', then the coefficient of mutual inductance between the two loops is:

(1) $\frac{\mu_0}{4} 8\sqrt{2}\frac{b^2}{a}$ (2) $\frac{\mu_0}{4}\frac{8\sqrt{2}}{b}$ (3) $\frac{\mu_0}{4}\frac{8\sqrt{2}}{a}$ (4) $\frac{\mu_0}{4} 8\sqrt{2}\frac{a^2}{a}$

Ans. (4)

Sol.



$$B \quad 4 \quad \frac{{}_0 i}{4 \quad \frac{b}{2}} \sqrt{2} \qquad\qquad B \quad \frac{2\sqrt{2} \quad {}_0 i}{b}$$

$$\Rightarrow \qquad \phi = MI \qquad\qquad \Rightarrow \qquad BA = MI$$

$$\frac{2\sqrt{2} \quad {}_0 i}{b} \quad a^2 \quad Mi \qquad\qquad B \quad \frac{2\sqrt{2} \quad {}_0 i}{b}$$

20. Which of the following equations is dimensionally incorrect?

Where t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, v = volume, p = pressure, W = work done, Γ = torque, $\in$ = permittivity, E = electric field, J = current density, L = length.

(1) J $\frac{E}{t}$ (2) v $\frac{pa^4}{8 \quad L}$ (3) h $\frac{2s\cos}{rg}$ (4) W = $\Gamma\theta$

Ans. (2)

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. If the sum of the heights of transmitting and receiving antennas in the line of sight of communication is fixed at 160 m, then the maximum range of LOS communication is _____km

(Take radius of Earth = 6400 km)

Ans. (64)

Sol. $h_r + h_t = 160$ (Given)

$$LOS \quad \sqrt{2R}\sqrt{h_r} \quad \sqrt{h_t}$$

$$LOS \quad \sqrt{2R}\sqrt{h_r} \quad \sqrt{160 \quad h_r}$$

$$\frac{dLOS}{dh_r} \quad 0 \quad \sqrt{2R}\frac{1}{2\sqrt{h_r}} \quad \frac{1}{2\sqrt{160 \quad h_r}} \quad 0$$

$h_r = h_t = 80$ m

$$LOS \quad \sqrt{2R}\sqrt{80} \quad \sqrt{80} \quad 2\sqrt{2 \quad 6400 \quad 10^3 \quad 80} \quad 64 \text{ Km}$$

2. A wire having a linear mass density 9.0×10^{-4} kg/m is stretched between two rigid supports with a tension of 900 N. the wire resonates at a frequency of 500 Hz. The next higher frequency at which the same wire resonates is 550 Hz. The length of the wire is_____m.

Ans. (10)

Sol. $f_n \quad \frac{nv}{2\ell} \quad 500$

$$f_{n\ 1} \quad \frac{(n \quad 1)v}{2\ell} \quad 550$$

$$\frac{n \quad 1}{n} \quad \frac{11}{10}$$

Thus, $n = 10$

$$\text{Thus,} \quad \ell \quad \frac{nv}{2f_n} \quad v \quad \sqrt{\frac{T}{}}$$

$$\frac{10}{2 \quad 500} \quad \sqrt{\frac{900}{9 \quad 10^{-4}}} \quad 10\,m$$

Ans. 10.00

3. A particles of mass 1 kg is hanging from a spring of force constant 100 Nm^{-1}. The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T. The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$. The value of x is_____.

Ans. (8)

Sol. KE = PE

$\frac{1}{2}k(A^2 \quad x^2) \quad \frac{1}{2}kx^2$

$x^2 \quad \frac{A^2}{2}$

$x \quad \frac{A}{\sqrt{2}}$

$y = A\sin\omega t$

$\frac{A}{\sqrt{2}} \quad A\sin \quad t$

$\sin\frac{}{4} \quad \sin \quad t$

$t \quad \frac{}{4}; t \quad \frac{T}{8}$. Thus x = 8

4. The electric field in an electromagnetic wave is given by E = (50 NC^{-1}) sin ω (t–x/c). The energy contained in a cylinder of volume V is 5.5 × 10^{-12} J. the value of V is _____cm^3.

(given $\in_0$ = 8.8 × 10^{-12} C^{2-} N^{-1} m^{-2})

Ans. (500)

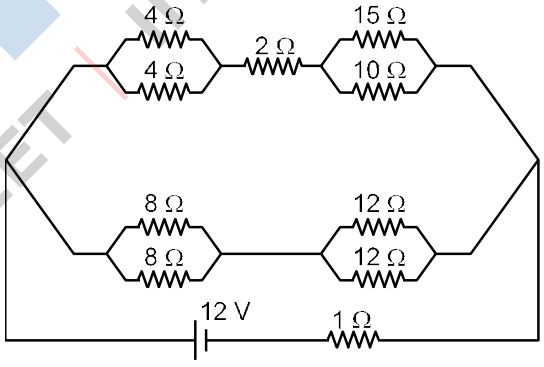
Sol. Energy density $\frac{1}{2} \quad _0 E^2$

Energy $\frac{1}{2} \quad _0 E^2 \quad$ Volume

$5.5 \quad 10^{\ 12} \quad \frac{1}{2} \quad 8.8 \quad 10^{\ 12} \quad 50^2 \quad V$

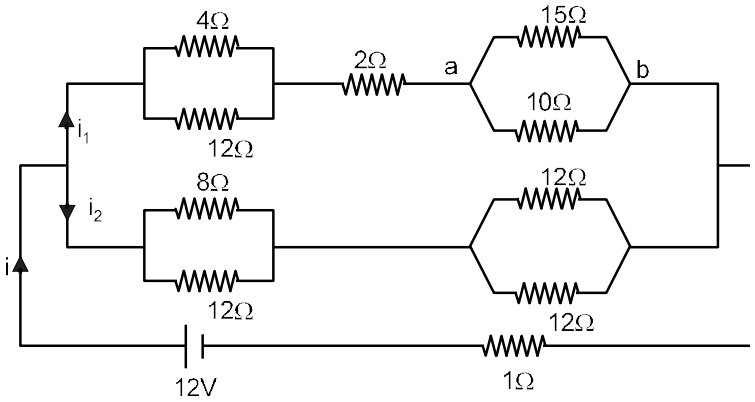
V = 500 cm^3

5. The voltage drop across 15 Ω resistance in the given figure will be _____v.

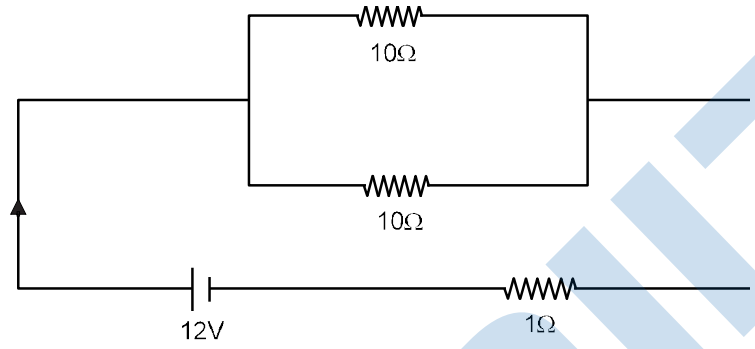


Ans. (6)

Sol.



equivalent circuit



$i \quad \frac{12}{6} \quad 2A$

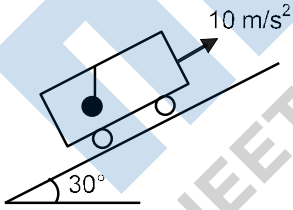
$i_1 = i_2 = i/2 = 1$ A

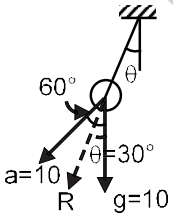
$V_{15\Omega} = i_1 \times R_{ab} = 1 \times 6 = 6V$

6. A car is moving on a plane inclined at 30° to be horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of car. The angle in degrees which the string makes with the vertical is_____. (Given = 10 ms^{-2})

Ans. 30

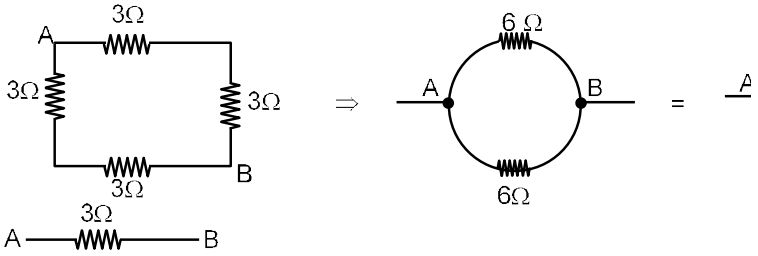
Sol.



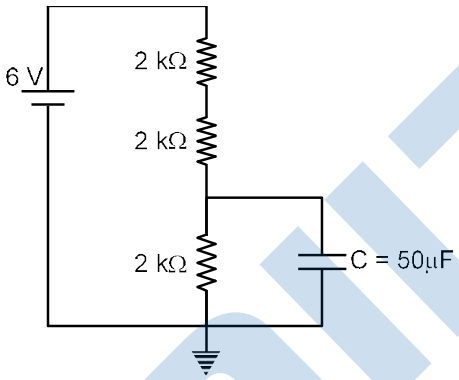


7. A square shaped wire with resistance of each side 3 Ω is bent to form a complete circle. The resistance between two diametrically opposite points of the circle in unit of Ω will be_____.

Ans.



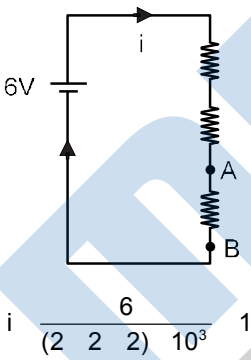
8. A capacitor of 50 μF is connected in a circuit as shown in figure. The charge on the upper plate of the capacitor is ____μC.



Ans. 10

Sol. At steady state current through branch of capacitor is zero. Potential difference across capacitor is 2V.

So, q = 50 × 2 = 100 μC



$i = \frac{6}{(2+2+2)\times 10^3} = 1\times 10^{-3}$ Amp

$V_{A,B}$ = potential across 50 μF

$V_{A,B} = i \times 2 \times 10^3 = 1 \times 10^{-3} \times 2 \times 10^3 = 2$ volt

Charge on 50 μF

$q = CV = 50 \times 10^{-6} \times 2 \Rightarrow 100 \times 10^{-6}$ C = 100 μC

9. When a rubber ball is taken to a depth of _____ m in deep sea, its volume decrease by 0.5%.

(The bulk modulus of rubber = 9.8×10^8 Nm^{-2}, Density of sea water = 10^3 kgm^{-3}, g = 9.8 m/s^2)

Ans. 500

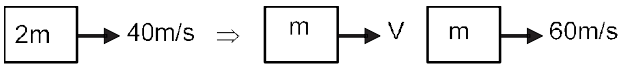
Sol. $\frac{dv}{v} \quad \frac{0.5}{100} \quad 0.005$

$B \quad \frac{\frac{dp}{dv}}{v} \quad \frac{\frac{pgh}{dv}}{v}$

$9.8 \quad 10^{8} \quad \frac{10^{3} \quad 9.8 \quad d}{5 \quad 10^{3}} \quad d \quad 500\,m$

10. A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, them the fractional change in the kinetic energy will be x : 4 where x = _____

Ans. 1

Sol. 

Initial Final

$2m \times 40 = 60 \times m + mv$

$V = 20$ m/s

Now $KE \quad KE_f \quad KE_i \quad \frac{1}{2}m \quad (20)^2 \quad \frac{1}{2}m \quad (60)^2 \quad \frac{1}{2}2m \quad (40)^2$

$KE \quad \frac{1}{2}m \quad (800) \quad KE_i \quad \frac{1}{2}2m \quad (40)^2 \quad \frac{1}{2}m \quad 3200$

Fractional change $\frac{KE}{KE_i} \quad \frac{1}{4}$

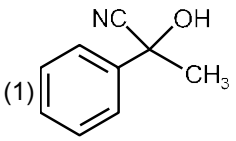
PART B : CHEMISTRY

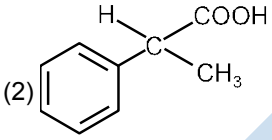
Single Choice Type

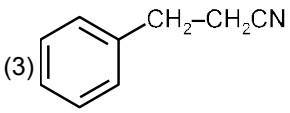
This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

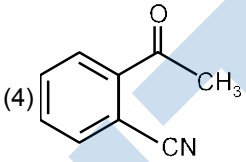
1. The structure of product C, formed by the following sequence of reactions is :

$CH_3COOH \xrightarrow{SOCl_2} A \xrightarrow[AlCl_3]{Benzene} B \xrightarrow[OH]{KCN} C$

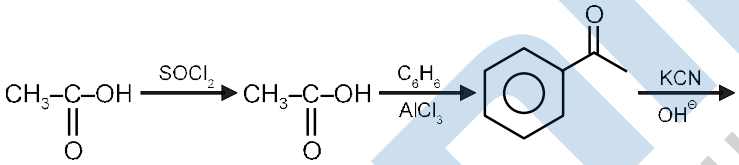
(1) 

(2) 

(3) 

(4) 

Ans. (1)

Sol. 

2. Which one of the following lanthanides exhibits +2 oxidation state with diamagnetic nature.

(Given Z for Nd = 60, Yb = 70, La = 57, Ce = 58)

(1) Yb (2) La (3) Nd (4) Ce

Ans. (1)

Sol. La^{2+} (Z = 57) = $5d^1$

Ce^{2+} (Z = 58) = $4f^2$

Nd^{2+} (Z = 60) = $4f^4$

Yb^{2+} (Z = 70) = $4f^{14}$

Yb^{2+} Contain no unpaired electron. So it is diamagnetic and Colourless.

3. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Aluminium is extracted from bauxite by the electrolysis of molten mixture of Al_2O_3 with cryolite.

Reason (R) : The oxidation state of Al in cryolite is +3

In the light of the above statements, choose the most appropriate answer from the options given below

(1) (A) is false but (R) is true.

(2) (A) is true but (R) is false.

(3) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(4) Both (A) and (R) are correct and (R) is not the correct explanation of (A).

Ans. (4)

Sol. In electrolytic reduction of Al_2O_3 we use cryolite to increase conductivity and decrease melting point. Cryolyte is Na_3AlF_6, Oxidation number of Al is +3

4. Given below are two statements :

Statement I : The process of producing syn-gas is called gasification of coal.

Statement II : The composition of syn-gas is CO + CO_2 + H_2 (1 : 1 : 1)

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are true. (2) Statement I is false but Statement II is true.

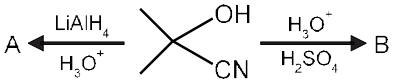
(3) Both Statement I and Statement II are false. (4) Statement I is true but Statement II is false.

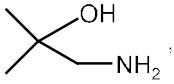
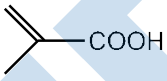
Ans. (4)

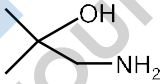
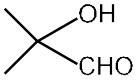
Sol. $\Rightarrow$ Mixture of CO and H_2 is called water gas or syn gas

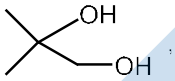
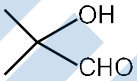
$\Rightarrow$ Syn gas is produced by coal gasification $C(s) \quad H_2O(g) \xrightarrow{1270\,K} CO(g) \quad H_2(g)$

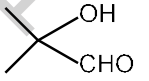
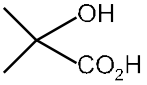
5. The major products A and B in the following set of reactions are :



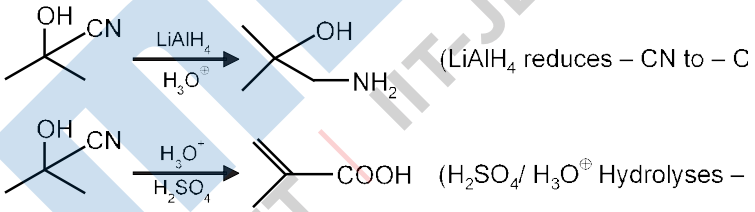
(1) A =  , B = 

(2) A =  , B = 

(3) A =  , B = 

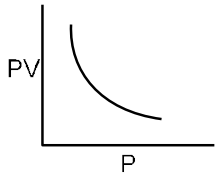
(4) A =  , B = 

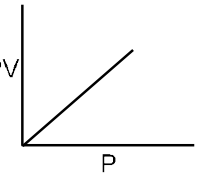
Ans. (1)

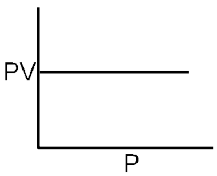
Sol. 

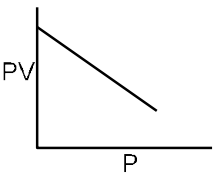
Alcohol undergoes elimination reaction in presence of H_2SO_4 / H_3O^+

6. Which one of the following is the correct PV vs P plot at constant temperature for an ideal gas? (P and V stand for pressure and volume of the gas respectively)

(1) 

(2) 

(3) 

(4) 

Ans. (3)

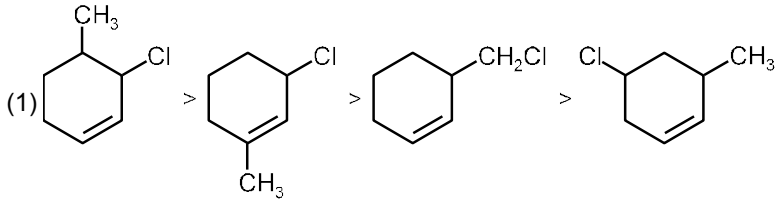
Sol. From ideal gas equation

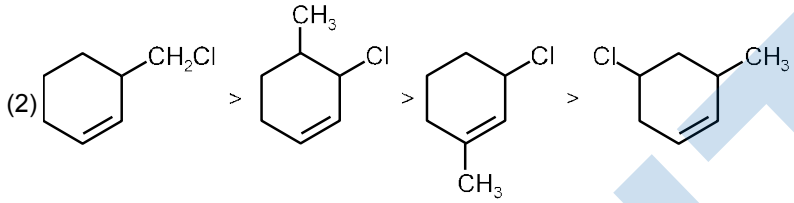
PV = nRT [At constant temperature for fixed amount of ideal gas]

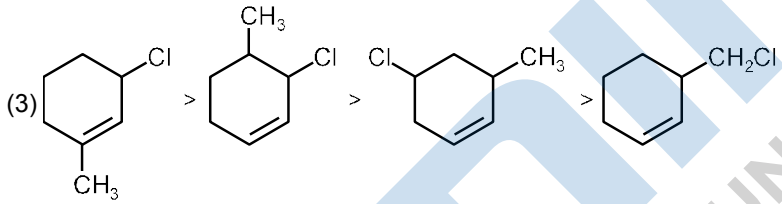
PV = constant

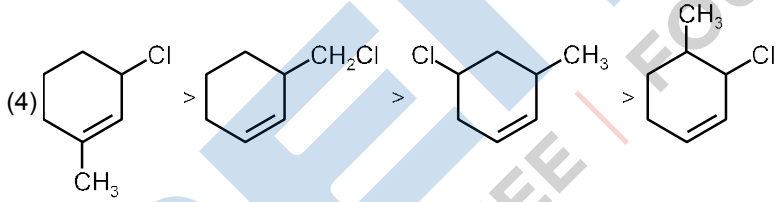
So, As P↑ ⇒ PV remain constant.

7. The correct order of reactivity of the given chlorides with acetate in acetic acids is :

(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. The given compounds in Acetate in Acetic acid solution reacts through S_N1 mechanism.

The stability of carbocation is directly proportion to reactivity via S_N1 mechanism.

8. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Treatment of bromine water with propene yields 1-bromopropan-2-ol.

Reason (R) : Attack of water on bromonium ion follows Markovnikov rule and results in 1-bromopropan-2-ol.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) (A) is false but (R) is true.

(2) Both (A) and (R) are true and (R) is the correct explanation of (A).

(3) (A) is true but (R) is false.

(4) Both (A) and (R) are true and (R) is not the correct explanation of (A).

Ans. (2)

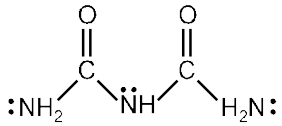
Sol. $CH_3 - CH = CH_2 \xrightarrow[H_2O]{Br_2} CH_3 - \underset{OH}{C} - \underset{Br}{CH_2}$

Markovnikov's rule of addition : In an electrophilic addition reaction to alkenes and alkynes the electrophile attacks in such a way that a more stable carbocation intermediate is formed on which nucleophile attacks in the next step.

9. The denticity of an organic ligand, biuret is :

(1) 6 (2) 2 (3) 3 (4) 4

Ans. (2)

Sol. Structure of biuret is 

in biuret donar atom are 2 nitrogen atom.

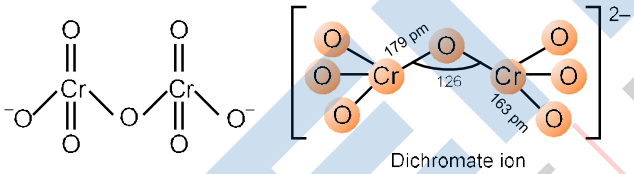
so it is a idented legend.

10. In the structure of the dichromate ion, there is a :

(1) non-linear unsymmetrical Cr–O–Cr bond. (2) non-linear symmetrical Cr–O–Cr bond.

(3) linear unsymmetrical Cr–O–Cr bond. (4) linear symmetrical Cr–O–Cr bond.

Ans. (2)

Sol. 

Dichromate ion

Non-linear, Symmetrical Cr – O – Cr bond

11. The major component/ ingredient of Portland Cement is :

(1) tricalcium aluminate (2) tricalcium silicate (3) dicalcium silicate (4) dicalcium aluminate

Ans. (2)

Sol. Portland cenment contain

(i) dicalcium silicate = 26%, (ii) Tricalcium silicate = 51%, (iii) Tricalcium aluminate = 11%

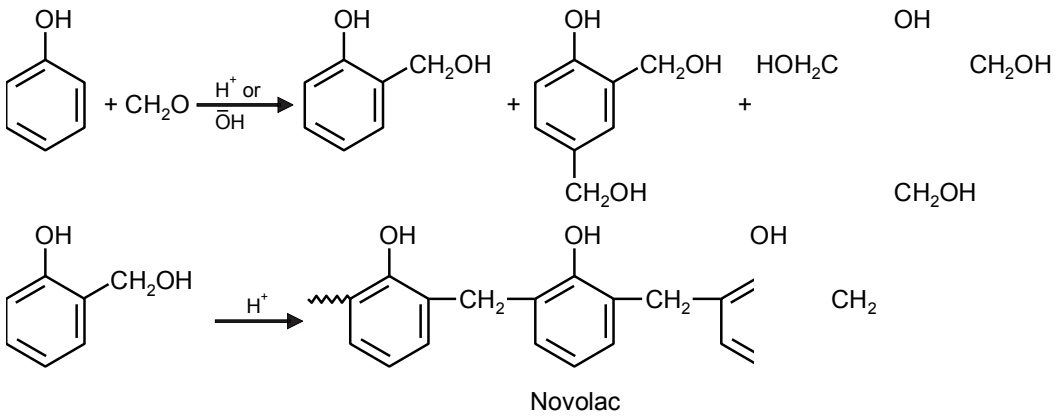
12. Monomer of Novolac is :

(1) phenol and melamine. (2) 3-Hydroxybutanoic acid.

(3) 1,3-Butadiene and styrene. (4) o-Hydroxymethylphenol.

Ans. (4)

Sol. Novalac : It is linear polymer of o-Hydroxymethylphenol.



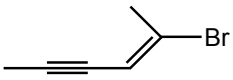
13. BOD values (in ppm) for clean water (A) and polluted water (B) are expected respectively

(1) A > 50, B < 27 (2) A < 5, B > 17 (3) A > 25, B < 17 (4) A > 15, B > 47

Ans. (2)

Sol. Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

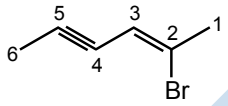
14. Choose the correct name for compound given below



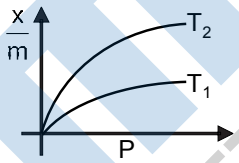
(1) (2E)-2-Bromo-hex-4-yn-2-ene (2) (2E)-2-Bromo-hex-2-en-4-yne

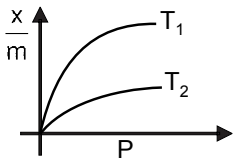
(3) (4E)-5-Bromo-hex-2-en-4-yne (4) (4E)-5-Bromo-hex-4-en-2-yne

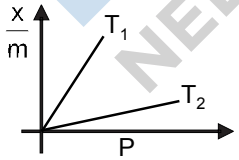
Ans. (2)

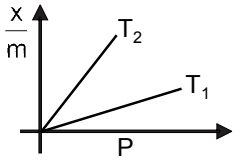
Sol.  ⟶ (2E)-2-Bromo-hex-2-en-4-yne

15. Select the graph that correctly describes the adsorption isotherms at two temperatures T_1 and T_2 ($T_1 > T_2$) for a gas : (x – mass of the gas adsorbed, m – mass of adsorbent, P – pressure)

(1) 

(2) 

(3) 

(4) 

Ans. (1)

Sol. On increase in temperature $\frac{x}{m}$ decreases.

16. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : A simple distillation can be used to separate a mixture of propanol and propanone.

Reason (R) : Two liquids with a difference of more than 20°C in their boiling points can be separated by simple distillations.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) (A) is false but (R) is true.

(2) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(3) Both (A) and (R) are correct and (R) is not the correct explanation of (A).

(4) (A) is true but (R) is false.

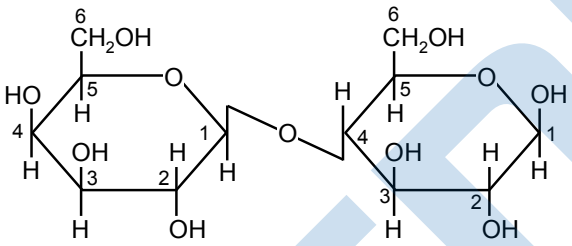
Ans. (2)

17. Which one of the following compounds contains β-C_1–C_4 glycosidic linkage ?

(1) Maltose (2) Sucrose (3) Lactose (4) Amylose

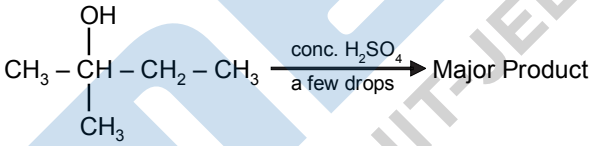
Ans. (3)

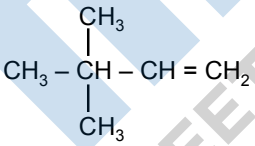
Sol. Structure of Lactose ($C_{12}H_{22}O_{11}$)

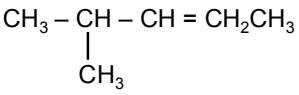


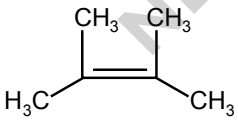
The linkage is between C-1 of β-Galactose and C-4 of β-Glucose.

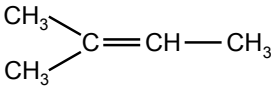
18. The major product formed in the following reactions is :



(1) 

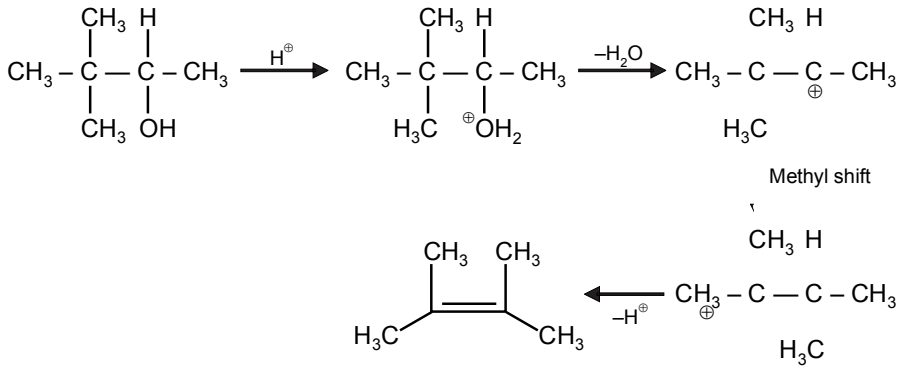
(2) 

(3) 

(4) 

Ans. (3)

Sol.



19. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Metallic character decreases and non-metallic character increases on moving from left to right in a period.

Reason (R) : It is due to increase in ionisation enthalpy and decrease in electron gain enthalpy, when one moves from left to right in a period.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(2) (A) is false but (R) is true.

(3) (A) is true but (R) is false.

(4) Both (A) and (R) are correct and (R) is not the correct explanation of (A).

Ans. (3)

Sol. On moving L $\rightarrow$ R $\Rightarrow$ Metallic Character $\downarrow$ non Metallic Character $\uparrow$

It is due increase in both IE & EA.

20. Which one of the following 0.10 M aqueous solutions will exhibit the largest freezing point depression ?

(1) hydrazine (2) glucose (3) $KHSO_4$ (4) glycine

Ans. (3)

Sol. $\Delta T_f = i\, k_f\, m$

As m is same for all electrolyte so

$\Delta T_f \alpha\, i\, m$

Greater the value of i greater is depression in freezing point

For $KHSO_4$ i = 3

And for Glucose, Ethylele Glycol & glycerol i = 1

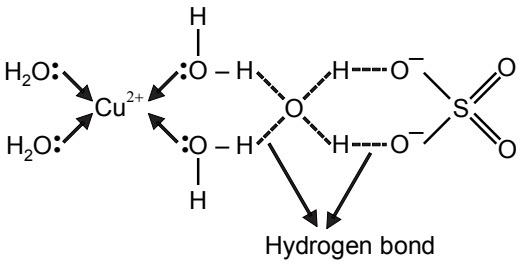
Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The number of hydrogen bonded water molecule(s) associated with stoichiometry $CuSO_4 \cdot 5H_2O$ is_____

Ans. (1)

Sol.



2. For a first order reaction, the ratio of the time for 75% completion of a reaction to the time for 50% completion is _____

Ans. (2)

Sol. $T \quad \frac{2.303}{k} \log \frac{100}{100 \quad x\%}$

$T_{75\%} \quad \frac{2.303}{k} \log \frac{100}{25}$

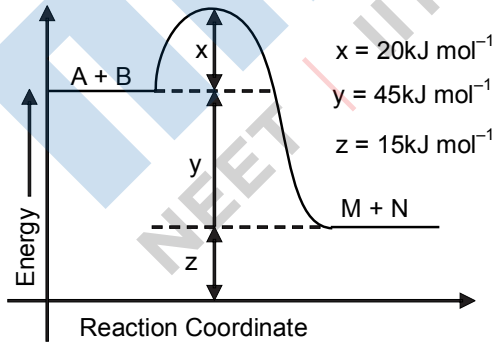
$T_{50\%} \quad \frac{2.303}{k} \log \frac{100}{50}$

$\frac{T_{75\%}}{T_{50\%}} \quad \frac{\log 4}{\log 2} \quad 2$

3. According to the following figure, the magnitude of the enthalpy change of the reaction

$A + B \rightarrow M + N$ in kJ mol^{-1}

is equal to _____.



Ans. (45)

Sol. Here $\Delta H_{rxn.} = y = 45$ Kj/mole

4. The number of halogen/(s) forming halic (V) acid is _____

Ans. (3)

Sol. Except F all other halogen form XO_3 ion like ClO_3, BrO_3 IO_3

5. The total number of reagents from those given below, that can convert nitrobenzene into aniline is_____.

I. Sn — HCl II. Sn — NH_4OH III. Fe — HCl

IV. Zn — HCl V. H_2 — Pd VI. H_2 — Raney Nickel

Ans. (5.00)

Sol. Nitrobenzene converted into Aniline by – Sn + HCl, Fe + HCl, H_2/Pd, H_2/Raney Ni.

6. The molarity of the solution prepared by dissolving 6.3 g of oxalic acid ($H_2C_2O_4 \cdot 2H_2O$) in 250 mL of water in mol L^{-1} is $x \times 10^{-2}$. The value of x is _____

[Atomic mass : H: 1.0, C : 12.0, O : 16.0]

Ans. (20)

Sol. $$M \quad \frac{W_{solute} \quad 1000}{GMM_{solute} \quad V_{ml}}$$

$$\frac{6.3 \quad 1000}{126 \quad 250} \quad 0.2M$$

$= 20 \times 10^{-2}$

So $X = 20$

7. Consider the sulphides HgS, PbS, CuS, Sb_2S_3, As_2S_3 and CdS. Number of these sulphides soluble in 50% HNO_3 is _____

Ans. (4)

Sol. CdS, PbS, As_2S_3, and CuS, are soluble in 50% HNO_3 while HgS and Sb2S3 are insoluble in 50% HNO_3 but HgS soluble in aqua regia.

8. A_3B_2 is a sparingly soluble salt of molar mass M (g mol^{-1}) and solubility x gL^{-1}. The solubility product satisfies $K_{sp} \quad a \left(\frac{x}{M}\right)^5$. The value of a is _____.

Ans. (108)

Sol. Solubility $\frac{x}{m}$ mole/ lit

$$A_3B_2\,(s) \rightleftharpoons 3A^{2+}(aqs) + 2B^{3-}(aqs)$$

$$\frac{3x}{M} \qquad 2\,\frac{x}{M}$$

$$K_{sp} \quad \left(\frac{3x}{M}\right)^3 \left(\frac{2x}{M}\right)^2$$

$$(3)^3(2)^2 \left(\frac{x}{m}\right)^5$$

$$27 \quad 4\left(\frac{x}{m}\right)^5$$

$$108\left(\frac{x}{m}\right)^5 \quad a\left(\frac{x}{m}\right)^5$$

So a = 108 Ans. 108

9. Consider the following cell reaction

$$Cd(s) \quad Hg_2SO_4(s) \quad \frac{9}{5}H_2O(\ell) \rightleftharpoons CdSO_4 \quad \frac{9}{5}H_2O(s) \quad 2Hg(\ell)$$

The value of E^0_{cell} is 4.315 V at 25°C. If $\Delta H^\circ = -825.2$ kJ mol^{-1}, the standard entropy change ΔS° in J K^{-1} is _____.

[Given : Faraday constant = 96487 C mol^{-1}]

Ans. (25)

Sol. For given cell reaction n = 2

and $\Delta G^\circ = -nFE^\circ_{cell}$, $\Delta G^\circ = \Delta H^\circ - T\Delta S$

Then $\Delta H^\circ - T\Delta S^\circ = -nF\,E^\circ_{cell}$

$\Delta H^\circ = 825.2\times10^3$ J/mole

T = 298 K

E°_{cell} = 4.315 V

F = 96485 Cb

$$S \quad \frac{nFE^\circ_{cell} \quad H}{T}$$

$$S \quad \frac{2 \quad 96487 \quad 4.315 \quad (\ 825.2 \quad 10^3)}{298} \quad \frac{832.682 \quad 10^3 \quad 825.2 \quad 10^3}{298} \quad \frac{7482}{298} \quad 10^3$$

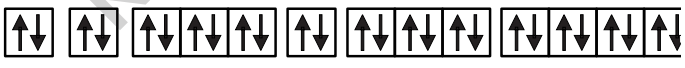
ΔS° = 25.1 J/k

10. Ge (Z = 32) in its ground state electronic configuration has x completely filled orbitals with $m_\ell = 0$. The value of x is _____

Ans. (7)

Sol. $_{32}Ge = 1s^2\ 2s^2\ 2p^6\ 3s^3\ 3p^6\ 3p^6\ 4s^3\ 4p^2$

ℓ 0 0 1 0 1 1 0 1



$m_{(\ell)}$ 0 0 –1 0 +1 0 –1 0 +1 –2 –1 0 +1 +2 0 –1 0

So in Ge (Z = 32) in its ground state electronic configuration has 7 completely filled orbitals with $m_\ell = 0$

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Let $\vec{a}$ & $\vec{b}$ be two vectors such that $|2\vec{a} \quad 3\vec{b}| \quad |3\vec{a} \quad \vec{b}|$ and the angle between $\vec{a}$ & $\vec{b}$ is 60º If $\frac{1}{8}$ is a unit vector, then $|\vec{b}|$ is equal to :

(1) 5 (2) 4 (3) 8 (4) 6

Ans. (1)

Sol. $|\vec{a}| \quad 8$

$|2\vec{a} \quad 3\vec{b}|^2 \quad |3\vec{a} \quad \vec{b}|^2$

$4|\vec{a}|^2 \quad 9|\vec{b}|^2 \quad 12|\vec{a}||\vec{b}|\cos 60 \quad 9|\vec{a}|^2 \quad |\vec{b}|^2 \quad 6|\vec{a}||\vec{b}|\cos 60$

$5|\vec{a}|^2 \quad 8|\vec{b}|^2 \quad 6|\vec{a}||\vec{b}|\cos 60$

$5(64) \quad 8|\vec{b}|^2 \quad 6(8) \quad |\vec{b}|\frac{1}{2}$

$|\vec{b}|^2 \quad 3|\vec{b}| \quad 40 \quad 0 \quad |\vec{b}| \quad 8 \quad |\vec{b}| \quad 5 \quad 0$

$|\vec{b}| \quad 5$

2. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^x - 6 = 0$ is :

(1) 1 (2) 4 (3) 2 (4) 0

Ans. (1)

Sol. $e^{4x} + 2e^{3x} - e^x - 6 = 0$

Let $e^x = t \qquad t > 0$

$f(t) = t^4 + 2t^3 - t - 6$

$f'(t) = 4t^3 + 6t^2 - 1$

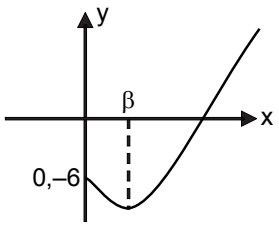
$f''(t) = 12t^2 + 12t > 0 \qquad (t > 0)$

$f'(t)$ is increasing function

Now $f'(0) = -1$

$f'(1) = 9$

So $f'(\beta) = 0$



3. If $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$, y(0) = 1 then y(1) is equal to :

(1) $\log_2(2e)$ (2) $\log_2(2 + e)$ (3) $\log_2(1 + e)$ (4) $\log_2(1 + e^2)$

Ans. (3)

Sol. $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y} = 2^x - \frac{2^y - 1}{2^y}$

$\frac{2^y dy}{2^y - 1} = 2^x dx$

Let $2^y - 1 = t \Rightarrow 2^y \ell n2\, dy = dt$

$\frac{dt}{t} \frac{1}{\ell n2} = 2^x dx$

$\frac{\ell nt}{\ell n2} = \frac{2^x}{\ell n2} + \frac{C}{\ell n2}$

$\ell nt = 2^x + C$

when x = 0, y = 1, C = –1

$\ell n(2^y - 1) = 2^x - 1$

$2^y - 1 = e^{2^x} - 1$

$2^y - 1 = e^{2^x} - 1$

when $x = 1 \Rightarrow 2^y = 1 + e^1 \Rightarrow y = \log_2(1 + e)$

4. Let $\ast, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p \ast \sim q) \Rightarrow (p \square q)$ is a tautology. Then :

(1) $\ast = \vee, \square = \vee$ (2) $\ast = \vee, \square = \wedge$ (3) $\ast = \wedge, \square = \vee$ (4) $\ast = \wedge, \square = \wedge$

Ans. (3)

Sol. $(p \wedge \sim q) \rightarrow (p \vee q)$

$= \sim (p \wedge \sim q) \vee (p \vee q)$

$= (\sim p \vee q) \vee (p \vee q)$

$= (\sim p \vee p) \vee (q \vee q) = t \vee q$

$= t$

5. Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes $3x - 2y + 4z - 7 = 0$ and $x + 5y - 2z + 9 = 0$, be $\alpha x + \beta y + \gamma z + 3 = 0$, that $\alpha + \beta + \gamma$ is equal to :

(1) 15 (2) 23 (3) –23 (4) –15

Ans. (3)

Sol. Equation of plane $\Rightarrow P_1 + \lambda P_2 = 0$

$\Rightarrow$ $(3x - 2y + 4z - 7) + \lambda (x + 5y - 2z + 9) = 0$

Passing through the point (1, 4, –3)

$\Rightarrow$ $(3 - 8 - 12 - 7) + \lambda (1 + 20 + 6 + 9) = 0$

$\frac{24}{36}$ $\frac{2}{3}$

Now, Equation of plane is $3 \frac{2}{3} x \quad y \quad 2 \frac{10}{3} \quad z \; 4 \frac{4}{3} \quad 7 \quad 6 \quad 0$

$\Rightarrow$ $-11x - 4y - 8z + 3 = 0$

So, $\alpha + \beta + \gamma = -23$

6. The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves ?

(1) $x^2 + y^2 = 169$ (2) $144x^2 + 25y^2 = 3600$ (3) $25x^2 + 12y^2 = 3600$ (4) $x^2 + y^2 = 60$

Ans. (2)

Sol. $\frac{x\cos}{5} \quad \frac{y\sin}{12} \quad 1$ .....(i)

$\Rightarrow$ Let ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$

$\Rightarrow$ Equation of tangent at point $(a\cos\theta, b\sin\theta)$

$\frac{xa\cos}{a^2} \quad \frac{yb\sin}{b^2} \quad 1$

$\frac{x\cos}{a} \quad \frac{y\sin}{b} \quad 1$ .....(ii)

Using equation (i) and (ii) $\Rightarrow$ a = 5 b = 12

Equation of ellipse is $\frac{x^2}{25} \quad \frac{y^2}{144} \quad 1$

7. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18m away from the base of the pole, then the height of the pole (in meters) is :

(1) $12\sqrt{15}$ (2) $6\sqrt{10}$ (3) $8\sqrt{10}$ (4) $12\sqrt{10}$

Ans. (4)

Sol. $\tan \quad \frac{x}{6}$

$$\tan 2 \quad \frac{10x}{18} \quad \frac{5x}{9}$$

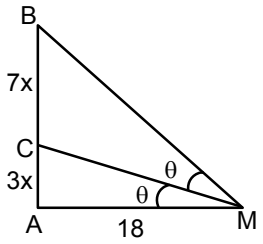
$$\frac{5x}{9} \quad \frac{\frac{x}{3}}{1 \quad \frac{x^2}{36}}$$

$$5 \quad \frac{3}{1 \quad \frac{x^2}{36}}$$

$$1 \quad \frac{x^2}{36} \quad \frac{3}{5}$$

$$\frac{x^2}{36} \quad \frac{2}{5}$$

$$x^2 \quad \frac{72}{5} \qquad x \quad \frac{6\sqrt{2}}{\sqrt{5}}$$



$\Rightarrow$ Length of pole $10x \quad \frac{60\sqrt{2}}{\sqrt{5}} \quad \frac{\sqrt{5}}{\sqrt{5}} \quad \frac{60\sqrt{10}}{5} \quad 12\sqrt{10}$

8. The integral $\int \frac{1}{\sqrt[4]{(x \quad 1)^3 (x \quad 2)^5}} dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{4}{3} \left(\frac{x \quad 1}{x \quad 2}\right)^{\frac{5}{4}} \quad C$ (2) $\frac{4}{3} \left(\frac{x \quad 1}{x \quad 2}\right)^{\frac{1}{4}} \quad C$ (3) $\frac{3}{4} \left(\frac{x \quad 2}{x \quad 1}\right)^{\frac{1}{4}} C$ (4) $\frac{3}{4} \left(\frac{x \quad 2}{x \quad 1}\right)^{\frac{5}{4}} \quad C$

Ans. (2)

Sol. $I \quad \int \frac{dx}{\sqrt[4]{(x \quad 1)^3 (x \quad 2)^5}}$

$$I \quad \int \frac{dx}{\left(\frac{x \quad 2}{x \quad 1}\right)^{5/4} (x \quad 1)^2}$$

Let $\frac{(x \quad 2)}{(x \quad 1)} \quad t \qquad \frac{3}{(x \quad 1)^2} dx \quad dt \quad I \quad \frac{\frac{1}{3} t^{1/4}}{\frac{1}{4}} \quad C \quad \frac{4}{3} \left(\frac{x \quad 1}{x \quad 2}\right)^{1/4} \quad C$

9. If p and q are the lengths of the perpendiculars from the origin on the lines, $x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$ and $x \sin \alpha + y \cos \alpha = k \sin 2\alpha$ respectively, then k^2 is equal to :

(1) $p^2 + 4q^2$ (2) $2p^2 + q^2$ (3) $p^2 + 2q^2$ (4) $4p^2 + q^2$

Ans. (4)

Sol. $L_1 : x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$

$$\frac{x}{\sin} \quad \frac{y}{\cos} \quad k \frac{\cos 2}{\sin 2}$$

$x\cos \quad y\sin \quad \frac{k}{2}\cos 2$

Perpendicular distance from (0, 0) is $p \quad \left|\frac{0 \quad 0 \quad \frac{k}{2}\cos 2}{\sqrt{\cos^2 \quad \sin^2}}\right|$

$$p^2 \quad \frac{k^2}{4}\cos^2 2$$

$L_2 : x \sin \alpha + y \cos \alpha - k \sin 2\alpha = 0$

Perpendicular distance from (0, 0) is $q \quad \left|\frac{0 \quad 0 \quad k\sin 2}{\sqrt{\sin^2 \quad \cos^2}}\right|$

$$\Rightarrow q^2 = k^2 \sin^2 2\alpha$$

$4p^2 + q^2 = k^2 (\sin^2 2\alpha + \cos^2 2\alpha)$

$4p^2 + q^2 = k^2$

10. The sum of 10 terms of the series $\frac{3}{1^2 \quad 2^2} \quad \frac{5}{2^2 \quad 3^2} \quad \frac{7}{3^2 \quad 4^2}$ ..... is :

(1) 1 (2) $\frac{143}{144}$ (3) $\frac{120}{121}$ (4) $\frac{99}{100}$

Ans. (3)

Sol. $\sum_{r\ 1}^{10} \frac{(2r \quad 1)}{r^2 \quad r \quad 1^2}$

$$\sum_{r\ 1}^{10} \frac{1}{r^2} \quad \frac{1}{r \quad 1^2}$$

$$\frac{1}{1^2} \quad \frac{1}{2^2} \quad \frac{1}{2^2} \quad \frac{1}{3^2} \quad \frac{1}{3^2} \quad \frac{1}{4^2} \quad .... \quad \frac{1}{10^2} \quad \frac{1}{11^2} \quad 1 \quad \frac{1}{121} \quad \frac{120}{121}$$

11. The length of the latus rectum of parabola, whose vertex and focus are on the positive x-axis at a distance R and S (>R) respectively from the origin, is :

(1) 2(S+R) (2) 4(S+R) (3) 4(S–R) (4) 2(S–R)

Ans. (3)

Sol. $y^2 = 4a (x - R)$

$y^2 = 4(S - R) (x - R)$

Length of LR = 4 (S– R)

12. The function $f(x) = |x^2 - 2x - 3| \cdot e^{|9x^2 \quad 12x \quad 4|}$ is not differentiable at exactly :

(1) one point (2) two points (3) four points (4) three points

Ans. (2)

Sol. $f(x) = |x^2 - 2x - 3| \ e^{|9x^2 \quad 12x \quad 4|}$

$$f(x)\begin{cases}(x^2+2x-3)\,e^{(3x-2)^2}; & x<1\\ (x^2+2x-3)\,e^{(3x-2)^2}; & 1\le x\le 3\\ (x^2+2x-3)\,e^{(3x-2)^2}; & 3<x\end{cases}$$

Now, clearly non differentially at $x = -1, 3$

13. $\lim\limits_{x\to 0}\dfrac{\sin^2 x-\cos^4 x}{x^4}$ is equal to :

(1) 4π (2) $2\pi^2$ (3) $4\pi^2$ (4) π^2

Ans. (3)

So. $\lim\limits_{x\to 0}\dfrac{\sin^2 x-1+\sin^2 x^2}{x^4}$

$$\lim_{x\to 0}\frac{\sin^2 x-1+\sin^4 x+2\sin^2 x}{x^4}\quad \lim_{x\to 0}\frac{\sin^2 x+2\sin^2 x-\sin^4 x}{x^4}$$

$$\lim_{x\to 0}\frac{\sin^2 x-2\sin^2 x-\sin^4 x}{x^4}\quad \lim_{x\to 0}\frac{\sin x-2\sin^2 x-\sin^4 x}{2\sin^2 x-\sin^4 x}\cdot\frac{2\sin^2 x-\sin^4 x}{x^4}$$

$$2\lim_{x\to 0}\frac{\sin x}{x}\cdot 2\cdot\sin^2 x\cdot 4$$

$$\frac{1}{x}\log_e\frac{1+\frac{x}{a}}{1-\frac{x}{b}},\quad x\neq 0$$

14. If the function $f(x)\begin{cases}k, & x=0\\ \dfrac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1}, & x\neq 0\end{cases}$ is continuous at $x = 0$, then $\dfrac{1}{a}\dfrac{1}{b}\dfrac{4}{k}$ is equal to :

(1) 4 (2) – 5 (3) 5 (4) –4

Ans. (2)

Sol. LHL $\lim\limits_{x\to 0}\dfrac{\log\left(1+\frac{x}{a}\right)}{x}-\dfrac{\log\left(1-\frac{x}{b}\right)}{x}$

$$\lim_{x\to 0}\frac{\log\left(1+\frac{x}{a}\right)-\log\left(1-\frac{x}{b}\right)}{x}$$

$$\lim_{x\to 0}\frac{\log\left(1+\frac{x}{a}\right)}{\frac{x}{a}}\cdot\frac{1}{a}-\frac{\log\left(1-\frac{x}{b}\right)}{\frac{x}{b}}\cdot\frac{1}{b}\quad \frac{1}{b}\quad \frac{1}{a}$$

RHL $\lim_{x \to 0} \frac{\cos^2 x \quad \sin^2 x \quad 1 \quad \sqrt{x^2 \quad 1} \quad 1}{\sqrt{x^2 \quad 1} \quad 1}$

$\lim_{x \to 0} \frac{2\sin^2 x}{x^2} \sqrt{x^2 \quad 1} \quad 1 \quad 4$

So, $\frac{1}{b} \quad \frac{1}{a} \quad k \quad 4 \qquad \frac{1}{a} \quad \frac{1}{b} \quad \frac{4}{k} \quad 4 \quad 1 \quad 5$

15. Which of the following is not correct for relation R on the set of real numbers ?

(1) $(x, y) \in R \Leftrightarrow |x - y| \leq 1$ is reflexive and symmetric

(2) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric

(3) $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric

(4) $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive

Ans. (4)

Sol. $0 \quad |x \quad y| \quad 1$ is symmetric
$0 \quad |y \quad x| \quad 1$

let

$0 < |x - y| \leq 1$ .....(1)

$(1,2) \in R$ and $(2,3) \in R$ satisfy the equation (1)

but $(1,3) \notin R$ not satisfied

Hence $0 < |x - y| \leq 1$ is symmetric but not transitive relation

16. cosec18° is a root of the equation:

(1) $x^2 - 2x - 4 = 0$ (2) $4x^2 + 2x - 1 = 0$ (3) $x^2 - 2x + 4 = 0$ (4) $x^2 + 2x - 4 = 0$

Ans. (1)

Sol. $\sin 18 \quad \frac{\sqrt{5} \quad 1}{4}$

$\text{cosec}\,18 \quad \frac{4}{\sqrt{5} \quad 1} \quad \frac{\sqrt{5} \quad 1}{\sqrt{5} \quad 1} \quad \sqrt{5} \quad 1$

Hence equation $x \quad \sqrt{5} \quad 1$

$(x - 1)^2 = 5$

$x^2 - 2x - 4 = 0$

17. If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

(1) $a \quad \frac{1}{3}, b \quad \frac{7}{3}$ (2) $a \quad \frac{1}{3}, b \quad \frac{7}{3}$ (3) $a \quad \frac{1}{3}, b \quad \frac{7}{3}$ (4) $a \quad \frac{1}{3}, b \quad \frac{7}{3}$

Ans. (1)

Sol. $D \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & a \end{vmatrix} \quad 0 \quad 2 \quad 1 \quad 3a \quad 0 \quad a \quad \frac{1}{3}$

$D_1 \begin{vmatrix} 5 & 1 & 1 \\ 3 & 1 & 1 \\ b & 1 & a \end{vmatrix} \quad 2 \quad b \quad \frac{7}{3}$

$D_3 \begin{vmatrix} 2 & 5 & 1 \\ 1 & 3 & 1 \\ 1 & b & a \end{vmatrix} \quad \frac{7}{3} \quad b$

$D_3 \begin{vmatrix} 2 & 1 & 5 \\ 1 & 1 & 3 \\ 1 & 1 & b \end{vmatrix} \quad 7 \quad 3b$

For the system to have no solution $\quad b \quad \frac{7}{3}$

18. Three number are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to :

(1) $7 \quad \sqrt{3}$ (2) $7 \quad 3\sqrt{3}$ (3) $7 \quad 7\sqrt{3}$ (4) $7 \quad \sqrt{3}$

Ans. (4)

Sol. $\frac{a}{r}, a, ar, 3r^2$ G.P.

and $\frac{a}{r}, 2a, ar$ A.P.

$ar^2 = 3r^2$

$a = 3$

$4a \quad ar \quad \frac{a}{r}$

$4 \quad r \quad \frac{1}{r}$

$r^2 + 1 = 4r$

$r^2 \quad 4r \quad 1 \quad 0 \quad r \quad \frac{4 \quad \sqrt{12}}{2} \quad 2 \quad \sqrt{3}, 2 \quad \sqrt{3} \quad 2 \quad \sqrt{3}$ rejected

$d \quad 2a \quad \frac{a}{r} \quad a \; 2 \; \frac{1}{r} \quad 3 \; 2 \; \frac{1}{2 \quad \sqrt{3}} \quad 3 \; 2 \; 2 \quad \sqrt{3} \quad 3\sqrt{3}$

Hence $r^2 \quad d \quad 2 \quad \sqrt{3}^{\,2} \quad 3\sqrt{3} \quad 7 \quad \sqrt{3}$

19. Let f be a non-negative function in [0,1] and twice differentiable in (0,1).

If $\int_0^x \sqrt{1 - f'(t)^2}\,dt = \int_0^x f(t)\,dt$, $0 \le x \le 1$ and $f(0) = 0$, then $\lim_{x \to 0} \frac{1}{x^2} \int_0^x f(t)\,dt$:

(1) equal 1 (2) does not exist (3) equals 0 (4) equals $\frac{1}{2}$

Ans. (4)

Sol. $\int_0^x \sqrt{1 - f'(t)^2}\,dt = \int_0^x f(t)\,dt$

Diff w.r.t. x

$\sqrt{1 - f'(x)^2} = f(x)$

$\Rightarrow 1 - (f'(x))^2 = (f(x))^2 \quad \Rightarrow (f'(x))^2 = 1 - (f(x))^2$

$\left(\frac{dy}{dx}\right)^2 = 1 - y^2 \qquad \frac{dy}{dx} = \sqrt{1 - y^2} \qquad \frac{dy}{\sqrt{1 - y^2}} = dx$

by integration

$\sin^{-1} y = \pm x + C$

$\because y(0) = 0 \therefore C = 0$

but f is non negative

$\therefore \sin^{-1} y = x \Rightarrow y = \sin x$

Now $\lim_{x \to 0} \frac{1}{x^2} \int_0^x \sin t\,dt$

$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$

20. If $a_r = \cos\frac{2r}{9} + i\sin\frac{2r}{9}$, $r = 1,2,3,\ldots.$ $i = \sqrt{-1}$, then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to :

(1) $a_2a_6 - a_4a_8$ (2) $a_1a_9 - a_3a_7$ (3) a_9 (4) a_5

Ans. (2)

Sol. $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} e^{i\frac{2}{9}} & e^{i\frac{4}{9}} & e^{i\frac{6}{9}} \\ e^{i\frac{8}{9}} & e^{i\frac{10}{9}} & e^{i\frac{12}{9}} \\ e^{i\frac{14}{9}} & e^{i\frac{16}{9}} & e^{i\frac{18}{9}} \end{vmatrix} = e^{i\left(\frac{2}{9} + \frac{8}{9} + \frac{14}{9}\right)} \begin{vmatrix} 1 & e^{i\frac{2}{9}} & e^{i\frac{4}{9}} \\ 1 & e^{i\frac{2}{9}} & e^{i\frac{4}{9}} \\ 1 & e^{i\frac{2}{9}} & e^{i\frac{4}{9}} \end{vmatrix} = 0$

Now $a_1 = e^{i\frac{2}{9}}$ $a_9 = e^{i(2\pi)} = 1$

$a_1a_9 = a_3a_7 = e^{i\frac{20}{9}} = e^{i\frac{20}{9}} = 0$ $a_5 = e^{i\frac{10}{9}}$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. Let [t] denote the greatest integer $\le$ t. Then the value of $8.\int_{\frac{1}{2}}^{1}([2x] - |x|)dx$ is _____.

Ans. (5)

Sol. $I = \int_{\frac{1}{2}}^{1}([2x] - |x|)dx$

$$\int_{\frac{1}{2}}^{0}(1-x)dx + \int_{0}^{1/2}(0-x)dx + \int_{\frac{1}{2}}^{1}(1-x)dx$$

$$x - \frac{x^2}{2}\Big|_{\frac{1}{2}}^{0} - \frac{x^2}{2}\Big|_{0}^{1/2} + x - \frac{x^2}{2}\Big|_{\frac{1}{2}}^{1}$$

$$(0) - \frac{1}{2} + \frac{1}{8} - \frac{1}{8} + 1 - \frac{1}{2} - \frac{1}{2} + \frac{1}{8} - \frac{5}{8}$$

$$8\int_{\frac{1}{2}}^{1}([2x] - |x|)dx = 5$$

2. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _____ .

Ans. (576)

Sol. VOWELS

_ _ _ _ _ _

all the consonants never come together

= Total – all consonants come together

= 6! – 4! × 3! = 576

3. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p. then 98 p is equal to _____ .

Ans. (28)

Sol. E_1 = first unit is working

E_2 = second unit is working

$P(E_1) = 0.9 \qquad P(E_2) = 0.8$

$P(\bar{E}_1) = 0.1 \qquad P(\bar{E}_2) = 0.2$

$$P = \frac{0.8 \times 0.1}{0.1 \times 0.2 + 0.9 \times 0.2 + 0.1 \times 0.8} = \frac{8}{28}$$

So, 98 P 98 $\frac{8}{28}$ 28

4. If $\frac{3^6}{4^4}$ k is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$, then k is equal to ____.

Ans. (55)

Sol. $T_{r+1} = {}^{12}C_r \left(\frac{x}{4}\right)^{12-r} \left(\frac{12}{x^2}\right)^{r} = {}^{12}C_r \left(\frac{1}{4}\right)^{12-r} (-12)^r x^{12-r-2r}$

$\Rightarrow$ $12 - 3r = 0$

$\Rightarrow$ $r = 4$

So term independent from x is $T_5 = {}^{12}C_4 \frac{1}{4^8} 3^4 4^4 = {}^{12}C_4 \frac{3^4}{4^4} = 55 \frac{3^6}{4^4}$

5. The square of the distance of the point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-1}{6}$ and the plane $2x - y + z = 6$ from the point $(-1, -1, 2)$ is _____ .

Ans. (61)

Sol. Equation of line

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-1}{6}$

$\Rightarrow$ Let $P(2\lambda+1, 3\lambda+2, 6\lambda-1)$ is point of intersection of line and plane

$\Rightarrow$ $2(2\lambda+1) - 3\lambda - 2 + 6\lambda - 1 = 6$

$7\lambda = 7 \Rightarrow \lambda = 1$

Point P (3, 5, 5)

Square of distance from point $(-1, -1, 2)$ is

$= 4^2 + 6^2 + 3^2 = 61$

6. If the variable line $3x + 4y = \alpha$ lies between the two circles

$(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is _____ .

Ans. (165)

Sol. Line lies between the two circle

$(10 - \alpha)(31 - \alpha) < 0$

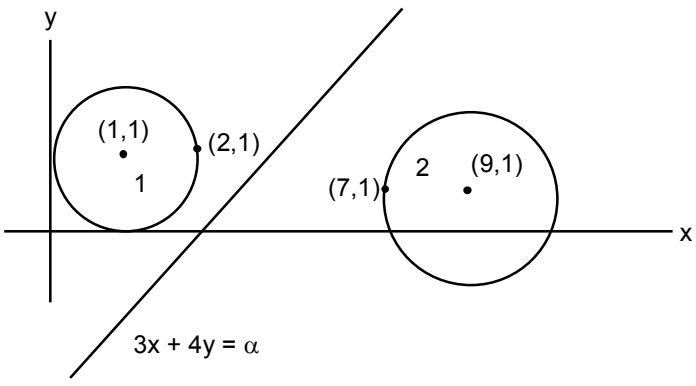
$10 < \alpha < 31$ .....(1)

As well as line can be tangent to the circle

$P_1 \Rightarrow$ (distance of (1,1) from line)

$P_2 \Rightarrow$ (distance of (9,1) from line)

$P_1 \geq r_1 \Rightarrow |7 - \alpha| \geq 10$



$\Rightarrow \alpha \in (-\infty,2] \cup [12,\infty)$ .....(2)

$P_2 \geq r_2 \Rightarrow |(31 - \alpha| \geq 10 \Rightarrow \alpha \in (-\infty,21] \cup [41, \infty)$ ..... (3)

$(1) \cap (2) \cap (3) \Rightarrow 12 \leq \alpha \leq 21$

sum of integer value of α = 12 + 13 + ... 21 $\frac{10}{2}(33)$ 165

7. A point z moves in the complex plane such that $\arg \frac{z \quad 2}{z \quad 2} \quad \frac{}{4}$, then the minimum value of $|z \quad 9\sqrt{2} \quad 2i|^2$ is equal to _____ .

Ans. (98)

Sol. $\arg \frac{z \quad 2}{z \quad 2} \quad \frac{}{4}$

$\arg \frac{x \quad iy \quad 2}{x \quad iy \quad 2} \quad \frac{}{4}$

$\arg \frac{x \quad iy \quad 2}{x \quad iy \quad 2} \frac{x \quad 2 \quad iy}{x \quad 2 \quad iy} \quad \frac{}{4}$

$\arg \frac{x^2 \quad y^2 \quad 4 \quad 4iy}{(x \quad 2)^2 \quad y^2} \quad \frac{}{4}$

$\frac{x^2 \quad y^2 \quad 4}{4y} \quad 1$

$x^2 + (y - 2)^2 = 8$ .....(1)

Equation (1) is represent circle, whose centre (0,2) and radius $2\sqrt{2}$

Minimum distance d = |AC – r|

given point A $9\sqrt{2},2$ $|Ac \quad r| \quad |9\sqrt{2} \quad 2\sqrt{2}| \quad 7\sqrt{2}$

$\Rightarrow$ square of distance $d^2 \quad 7\sqrt{2}^{\ 2} \quad 98$

8. If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1,2] and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1,2], then the value of |R – S| is _____ .

Ans. (2)

Sol. If $f'(x) = 2x + a \geq 0 \ \forall \ x \in [1,2] \Rightarrow a \geq -2x \ \forall \ x \in [1,2]$

$a_{min} = R = -2$

If $f'(x) \leq 0 \ \forall \ x \in [1,2] \Rightarrow 2x + a \leq 0 \Rightarrow a \leq -2x \ \forall \ x \in [1,2]$

$a \leq -4$

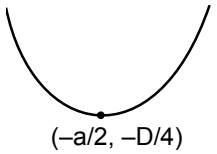
$a_{max} = S = -4$

$|R - S| = 2$

Alternate :

If f(x) is increasing for all $x \in [1,2]$ then $\frac{a}{2}$ 1 a 2 R 2

If f(x) is decreasing for $x \in [1,2]$, then $\frac{a}{2}$ 2 a 4 S 4



|R – S| = 2

9. If $x \quad (x) \quad \int_5^x (3t^2 \quad 2 \quad '(t))dt$, $x > -2$ and $\phi(0) = 4$, then $\phi(2)$ is _____ .

Ans. (4)

Sol. $x \quad (x) \quad \int_5^x (3t^2 \quad 2 \quad '(t))dt$

$\phi(0) = 4 \qquad x > -2$

$\Rightarrow x\,\phi'(x) + 1\phi(x) = 3x^2 - 2\,\phi'(x)$

$\Rightarrow (x + 2)\,\phi'(x) + \phi(x) = 3x^2$

I.F. $e^{\int \frac{1}{x \quad 2} dx} \quad x \quad 2$

$(x) \quad (x \quad 2) \quad (x \quad 2) \quad \frac{3x^2}{x \quad 2} \quad dx$

$\Rightarrow \phi(x) \cdot (x + 2) = x^3 + c$

$\Rightarrow \phi(0) = 4$

$c = 8$

$(2) \quad \frac{8 \quad 8}{4} \quad 4$

10. The mean 10 numbers

$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, \ldots\ldots$ is _____ .

Ans. 398

Sol. $\sum_{r\ 1}^{10} (3r \quad 4)(2r \quad 6)$

$\sum_{r\ 1}^{10} (6r^2 \quad 26r \quad 24) \quad 6 \sum_{r\ 1}^{10} r^2 \quad 26 \sum_{r\ 1}^{10} r \quad \sum_{r\ 1}^{10} 24 \quad 6 \frac{10.11.21}{6} \quad 26 \frac{10.11}{2} \quad 240$

$= 2310 + 1430 + 240 = 3980$

mean = 398